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"A Quarry What's It Worth?"

- Capital Investment Decision Making in the Quarrying Industry.

By

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A QUARRY - WHAT'S IT WORTH?

or "Capital Investment Decision Making in the Quarrying Industry"

July 2001

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INTRODUCTION

This paper is intended as a quarryman's introduction to economic evaluation. As the second title explains, the method is central to capital investment decision making. But why on earth would a quarry manager (or for that matter, other professionals in the industry such as engineers or geologists) need to know anything about economic analysis? If nothing more it may help in understanding personal finances, for example how to calculate compound interest. However, the real reason is that a broad understanding of the method is central to capital investment in say a new plant or even a new quarry. For example should my 30 year old crushing plant be replaced, upgraded or should I just accept low reliability and continue fixing it as it breaks down? Specifically the paper introduces the idea of "time value of money" and how it can be used to determine the value a quarry, the cost of a quarry product or the best quarry development.

But surely this is a job for the accountant or financier? Certainly not! This is a myth that still persists through much of New Zealand industry. The most difficult aspect of economic evaluation for a quarry is an understanding of the operation and making reasonable estimates of future costs and sales revenues, - tasks which are central to good quarry management. As the paper will show, the theory and calculations are quite straightforward. The methods have been widely used for a long time in other extractive industries such as mining and oil production. Sometimes referred to as financial analysis, the term "economic" is preferred as it focuses on the internal or project specific aspects of an investment.

This paper introduces the concept of "time value of money" with compounding interest. It then moves on to discounting - the inverse of interest, or what future income is worth today. Discounted cashflow (DCF) calculation, net present value (NPV) and rate of return are then discussed. Practical examples include valuation of a quarry for sale or purchase, establishing the cost of production, investment in a new plant or plant upgrade and how a quarry can be developed to get the best return.

Valuing a Quarry

Valuation of a quarry for sale and purchase can be attempted in a number of ways. In the end, the true value of something is only determined by what **"a willing purchaser is prepared to pay a willing vendor"**. Historic sales is one approach used in valuing other assets such as real estate and an estate agent or a "government valuation" will help establish what comparable properties have sold for recently.

The snag with quarries is that sales of quarries are not that common and how do you know that those quarries that have sold are comparable to yours (e.g. have the same life, environmental constraints, a plant with the same potential service life, cost of production and aggregate sales potential)? The solution is economic evaluation of your quarry as a going concern - as a business generating a cashflow.

The term evaluation describes this method and distinguishes it from valuation in general. The method is outlined in the following sections:

Cashflow

The first step in "evaluation as a going concern" is to estimate the cashflow - how cash will flow in and out of the business over the years. This entails estimating future sales volume and prices (Revenue) and operating costs (OpEx). Revenue minus operating costs is sometimes called operating profit (OpProfit). Once the annual operating profit (or loss) is calculated, any capital costs (CapEx) to sustain the operation are deducted and the Cashflow results.

EXAMPLE 1: VALUING A QUARRY AS A GOING CONCERN

An operating quarry is expected to trade for 15 years. Each year it is expected to sell 100,000 tonnes of aggregate at an average price of $22 per tonne. The average cost of production is estimated to be $15 per tonne. The mobile and fixed plant is in fair condition but needs the immediate injection of capital to the tune of $1 million so that with reasonable upkeep, it can be expected to last the 15 years. So what's this quarry worth?

To answer that, firstly we need to assemble the cashflow, a summary of the yearly total cash-in (+ve, income or revenue) and cash-out (-ve, costs or expenses) and then make allowance for the "time value of money". But first to prepare the annual cashflow projections over the 15 years (in millions) as shown in the following Table:

Table: Example 1 Cashflow

	Year 0	Year 1	Year 2	Year 3 > to > Year 14	Year 15
Production		100,000t	100,000t		100,000t
+ Revenue		$2.2	$2.2	>>>>>	$2.2
- OpEx		-$1.5	-$1.5	>>>>>	-$1.5
-CapEx	-$1.0				
= Cashflow	-$1.0	$0.7	$0.7	>>>>>	$0.7

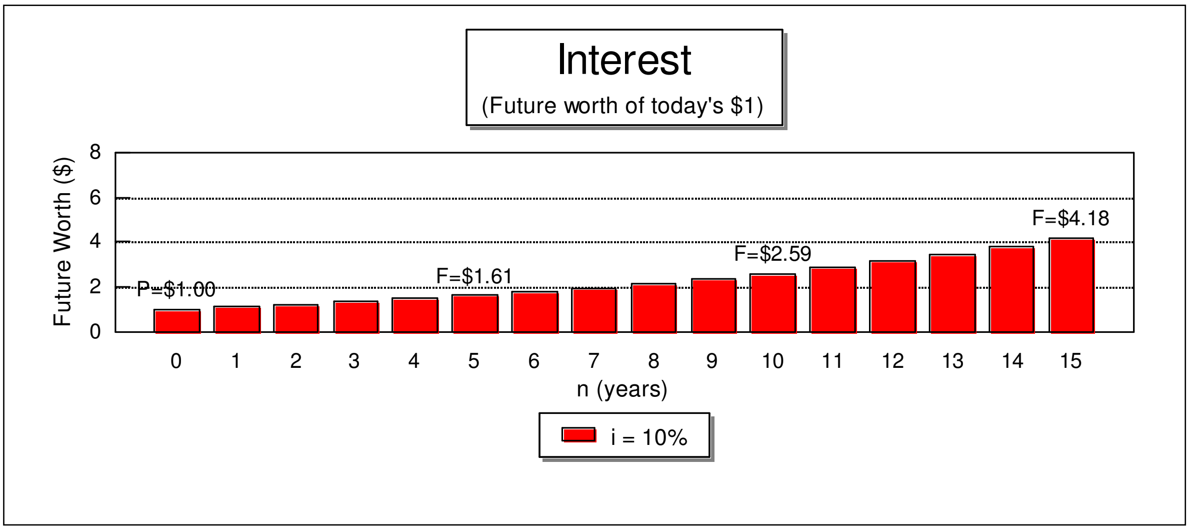
Note the use of the sign to distinguish income (+) and expenditure(-).

Having assembled the cashflow, the simplistic approach to answer "what its worth?" would be to use that basic accounting tool, the calculator and just add up the numbers. The problem is that we would be assuming that each dollar would be worth the same amount, irrespective of when it is spent or earned. This is definitely not the case, a dollar today is worth much more than a dollar in the future. This concept is called the "time value of money".

TIME VALUE OF MONEY

To understand how to account for the "time value of money" it is easiest to start with the familiar concept of compound interest:

The following graph illustrates that if we invest $1 today at 10% interest, next year it will have grown to $1.10 and if reinvested or compounded it will be worth $4.18 in 15 years time.



Although Bill Gates could sell you a "high tech" solution to run on your personal computer and tools such as spreadsheets are invaluable for economic evaluation, use of compound interest tables is a much better way to introduce and gain an understanding the "time value of money". Once the basis principles are understood, transferring the problem to a spreadsheet with built in "financial" functions is a breeze.

Examples of the interest tables at rates of 6%, 10%, 12% and 15% are included at the back of this paper. First a few definitions. In addition to the number of years (n) and the annual interest rate (i %), a very useful shorthand in bold text is used in the tables and the compound interest examples that follow:

Period (years)	**n**
Rate (%)	**i**
Present value ($)	**P**
Future value ($)	**F**
Uniform Annual amount	**A**

Compound Interest Calculation - $F/P_{i,n}$

The present worth or principal amount we invest at time zero is labelled **P** and the future value of our investment is **F**. The ratio for our compound interest problem is shown in the appropriate interest table at the back of the paper, in the column labelled as follows:

$$F/P_{i,n}$$

Looking up $F/P_{10\%,15yrs}$ in the tables we find that today's investment of $1, earning 10% compound, will have grown to $4.18 in 15 years (see also the interest graph above).

Mortgage or Capital Recovery $A/P_{i,n}$

What if we don't have any money to invest and we decide to borrow at 10% and repay the loan at the same amount over 15 years? If **A** is the regular amount that must repaid every year to clear the debt at a given rate of interest then the ratio of **A** as a proportion of the principal **P,** is found in the interest table, in the column labelled:

$$A/P_{i,n}$$

Looking up $A/P_{10\%,15yrs}$ in the tables, we find that every $1 we borrow at a rate of 10% per annum will cost us 13.15c every year for 15 years. This will cover interest and repayment of the principal.

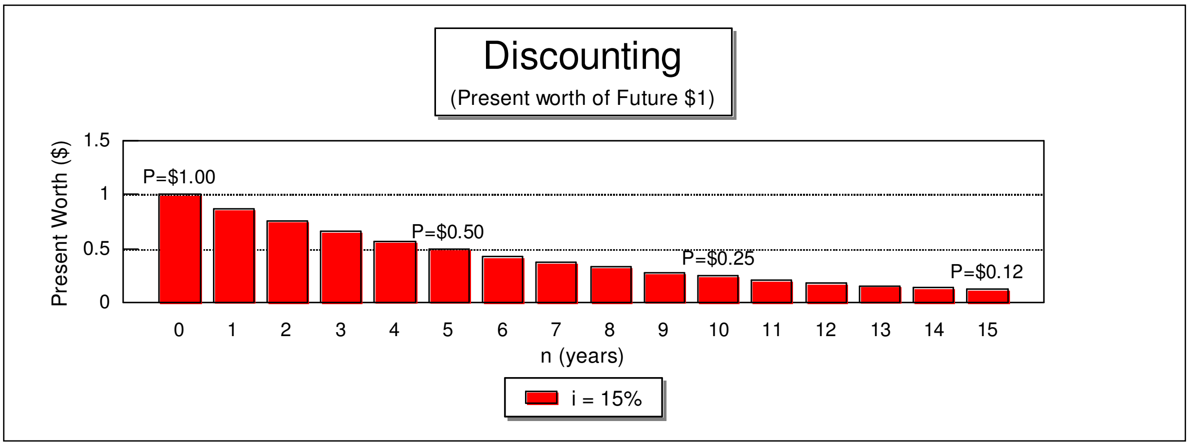
Returning from personal finances to our quarry problem: Suppose we want to work out the regular yearly amount **A** to repay and service the debt on the Capital Investment **P** of $1 million in Example 1, we can use the $A/P_{i,n}$ factor of 0.1315 which is now called a capital recovery factor, as follows:

$$\$1,000,000 \times 0.1315 = \$131,500 \text{ / year}$$

In this way when we are doing our monthly or annual costings, Capital Expenditure can be properly accounted for.

Discounting $P/F_{i,n}$

Interest involves calculating some future amount given a present sum. Discounting is the reverse process which involves calculating what some future amount **F** is worth today (**P**) as shown in the following graph:



Discounting is merely the inverse or reciprocal of interest and is used when we want to find the present value of some future dollar amount. The shorthand for the present worth or discount factor is **P/F.** This is equal to 1 divided by the compound interest factor: $F/P_{i,n}$ as follows:

$$\text{Present Worth Factor} = \frac{1}{\text{Future Worth Factor}} \quad \text{or} \quad P/F_{i,n} = \frac{1}{F/P_{i,n}}$$

Net Present Value (NPV)

The purchaser's required rate of return for new acquisitions is 15%. Multiplying each year's cashflow in Example 1 by the $P/F_{i,n}$ factor at a discount rate of 15% and adding up the amounts produces the Net Present Value (NPV). The NPV is what the quarry is worth as a going concern. The present worth uniform series factor ($P/A_{15\%15yrs}$ - the inverse of the capital recovery factor) is used to simplify the following calculation of the present worth of the regular annual cashflow or operating profit of $7 million in Example 1:

$$\begin{aligned} NPV &= (P/A_{15\%15yrs} \times \$0.7 \text{ million}) + (P/F_{i15\%\ 1yr} \times -\$1.0 \text{ million}) \\ &= (5.8474 \times \$0.7 \text{ million}) + (1 \times -\$1.0 \text{ million}) \\ &= \$3.093 \text{ million} \end{aligned}$$

So the short answer to What's it worth? for the purchaser is the Net Present Value or NPV of $3.093 million plus an allowance for other income or residual value in say the land or plant. The purchaser will also have to be prepared to inject the $1 million of capital. Of course a vendor will usually put a different slant on the calculation and produce a different NPV. Thus, when it comes to the actual sale, the true answer to What's it worth? is "What the willing purchaser is prepared to pay the willing vendor." Nevertheless, NPV is still the best price guide for both the purchaser and the vendor.

What Interest or Discount Rate (i%) Shall I Use?

The rate used in the compound interest problem depends upon a number of factors. The base lending rate is determined by the investment market and the Reserve Bank. The rate also depends on whether you are lending or borrowing. The borrowing rate will be the lending rate plus a margin the bank charges for managing the funds, plus a margin to cover the risk of the investment. To illustrate the risk component, a bank will lend at significantly lower rate, adding only a small margin for risk if it holds good security, than if the transaction is unsecured (through a credit card or loan shark).

The same applies to the investors in some venture or going concern. They will expect a required rate of return that covers the risk. The risk is greater for a NZ quarry than a fixed interest investment in the bank (hence add industry risk). A gold mine would probably be riskier than an quarry (add commodity risk). A quarry investment in say Colombia would be inherently riskier than a NZ counterpart (add country risk).

Required Rate of Return = Base Rate + Management Fee + Risks%

The important thing is that the investor's required rate of return is respected. In a public company the public shareholder's expectations determine the rate, or at least should determine the rate. In a company the answer to "does an investment stack up" is will an investment in say a new quarry or a plant upgrade meet the shareholder's expected rate of return? Irrespective of whether the investor is a shareholder in a public company, a ratepayer in a Council or a private or family investor in a private firm, the investors' expectation or what is called the "required rate of return" (ROR) is (or at least should be) the yardstick.

Which Rate of Return (ROR) Shall I Use?

The discount rate of 15% has been used for simplicity, a somewhat lower ROR would probably be used in most low risk quarry evaluations at the moment. Alternatively, to really confound, the required rate may be set as a sliding scale, varying by year (n) from say 12% to 15% to reflect long term uncertainty.

Sometimes other interest "rates" are used. For example a "real" rate is used with inflation adjusted dollars. Lower "after tax" rates are used when tax has been considered in preparing the cashflow. The important thing when preparing the cashflow is to make it as realistic as possible and use appropriate discount rates. The key is not to omit costs or "double dip" by charging twice for the same thing. For example, if a loss is being applied or in the case of tax relief for depreciation of a large capital investment, it is important that tax is handled properly and that the required discount rate is adjusted to compensate. (As in our personal finances a lower return is acceptable if tax has already been paid). I acknowledge that here some accounting input may be necessary!

The worked Example 1, can be repeated at different discount rates until the Net Present Value approaches zero. The discount rate that produces an NPV = 0 is called the Internal Rate of Return (IRR) for that investment. Although the IRR is a poor and overly simplistic test and not to be relied on solely, in general if the IRR is greater than the required rate of return then the investment is a "goer". IRR is determined by trial and error. In Example 1, the NPV is zero at a discount rate of approximately 70%. In other words the IRR is 70% which is well above most investor's expectations. But don't get too excited, the IRR will drop to 15% when a purchase price equal to the above NPV is paid and the capital for the plant upgrade is introduced!

EXAMPLE 2: PLANT UPGRADE

Turning to the next example "should my 30 year old crushing plant be replaced, upgraded or should I just accept low reliability and continue fixing it as it breaks down?" The solution involves careful preparation of a cashflow projections for each scenario (1) the "Do Nothing" - running of the existing plant, warts and all, (2) the plant upgrade and (3) a completely new installation. These are called "Service Producing Alternatives" and must provide an equivalent service (i.e. same life and output) if a fair comparison is to be made.

The careful preparation of the cashflows in this example is critical if the result is to be realistic. In the "Do Nothing" Option 1, the existing fix-it situation, the cost of repair, downtime, poor quality control and lost sales must be considered. Furthermore, the likelihood of having to replace the plant anyway, must be factored into the cashflow. Under the refurbishing Option 2, downtime for the work-over and the cost of stockpiling sufficient product to carry the operation and the hidden costs of the work must be considered. Under the new replacement Option 3, improved productivity is attractive but the possibility of overruns during design, construction and commissioning must be considered. It is in preparing the cashflows that an intimate knowledge of the operation as well as costs and plant design are necessary if a fair comparison of the service producing alternatives is to be made and the best option chosen. -Hence the job for the quarry manager! Unfortunately the details of this example are beyond the scope of this paper so we must move on to the results. Once the cashflows have been prepared they are discounted and the option with the best NPV (the lowest net present cost) is chosen:

Table: Plant Upgrade Comparison of NPV's

Option	Scenario	Net Present Cost NPV $ millions	Discounted processing cost per tonne
1	Existing situation - just fix-it	-$6.130 million	$9.00/t
2	Refurbish it.	-$5.945 million	$8.73/t
3	**Replace it.**	-$5.587 million	**$8.20/t**

Despite the high capital cost of the replacement plant, repair & maintenance costs are too expensive to justify keeping the old plant going, moreover, the existing plant is not consistently meeting specifications. Option 3 has the lowest discounted cost of processing, hence the decision is made to replace the plant.

EXAMPLE 3: COST OF PRODUCTION

When setting a sales price it is critically important that (a) the product remains competitive and (b) that true cost of production is considered. It is important that the capital investment that has been needed to sustain the cashflow is recovered by sales. Examples of such capital investment may have included the cost of securing resource consents, the cost of a new plant and the costs of pre-production development including overburden stripping. When stripping is carried out every few years rather than annually, it may be treated as a capital item. Such capital expenditure can be annualised by multiplying by the Capital Recovery Factor ($\mathbf{A/P}_{i,n}$) and allocated to the various products.

Discounted cost of production is also a key indicator when looking at different development options for the quarry. For example "should I develop the quarry in a particular direction even though this involves a heavy overburden stripping programme?" The solution is to establish the NPV for the various development options. The NPV can be divided by the sum of the discounted tonnes over the life of the project and the cheapest $/tonne option determined, as shown in the previous table.

CONCLUSIONS AND RECOMMENDATIONS

Unfortunately there has been insufficient time to present full worked examples and do any more than "whet the appetite". Nevertheless, it is hoped that the cursory treatment of the examples has added a certain reality to the paper.

The purpose of this paper has been to convince anyone involved in capital investment decision making that they can do basic economic evaluations. If the author has failed in this endeavour, the only request is to "give it another go". Try the methods out on the back of an envelope using the tables at the end of this paper. Try a spreadsheet. Talk to colleagues, even your accountant, financial advisor or quarry design consultant! Read and go on a course, but do not give up. The author learnt the skills relatively late in life after leaving college and has never regretted the effort. Please feel free to call the author if you need assistance.

CREDITS

Thanks are due to Frank Stermole for introducing the topic to a geologist / engineer in 1983. A special thanks to the many clients who have suffered endless brow beating and encouragement to adopt the methods.

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Compounding Interest Tables

Only a selection of rates has been used to prepare the tables that follow: 6% to reflect current bank savings, 10% bank lending, 12% and 15% for low risk rates of return. The reader that needs other rates should turn to a text or compute the values. The following notation is used:

Period (years)	**n**
Rate (%)	**i**
Present value ($)	**P**
Future value ($)	**F**
Uniform Annual amount	**A**

Single payment compound Interest factor	**F/Pi,n**
Present worth factor	**P/Fi,n**
Uniform series compound amount factor	**F/Ai,n**
Sinking fund factor	**A/Fi,n**
Capital recovery factor (mortgage)	**A/Pi,n**
Uniform series present worth factor	**P/Ai,n**

$$P/Fi,n = \frac{1}{(1+i)^n} = \frac{1}{F/Pi,n}$$

Compounding Interest Table

i = 12%

i = Rate %	n years	**F/Pi,n**	**P/Fi,n**	**F/Ai,n**	**A/Fi,n**	**A/Pi,n**	**P/Ai,n**
12%	0	1.000	1.000	0.000			
12%	1	1.120	0.893	1.000	1.0000	1.120	0.893
12%	2	1.254	0.797	2.120	0.4717	0.592	1.690
12%	3	1.405	0.712	3.374	0.2963	0.416	2.402
12%	4	1.574	0.636	4.779	0.2092	0.329	3.037
12%	5	1.762	0.567	6.353	0.1574	0.277	3.605
12%	6	1.974	0.507	8.115	0.1232	0.243	4.111
12%	7	2.211	0.452	10.089	0.0991	0.219	4.564
12%	8	2.476	0.404	12.300	0.0813	0.201	4.968
12%	9	2.773	0.361	14.776	0.0677	0.188	5.328
12%	10	3.106	0.322	17.549	0.0570	0.177	5.650
12%	11	3.479	0.287	20.655	0.0484	0.168	5.938
12%	12	3.896	0.257	24.133	0.0414	0.161	6.194
12%	13	4.363	0.229	28.029	0.0357	0.156	6.424
12%	14	4.887	0.205	32.393	0.0309	0.151	6.628
12%	15	5.474	0.183	37.280	0.0268	0.147	6.811
12%	16	6.130	0.163	42.753	0.0234	0.143	6.974
12%	17	6.866	0.146	48.884	0.0205	0.140	7.120
12%	18	7.690	0.130	55.750	0.0179	0.138	7.250
12%	19	8.613	0.116	63.440	0.0158	0.136	7.366
12%	20	9.646	0.104	72.052	0.0139	0.134	7.469
12%	21	10.804	0.093	81.699	0.0122	0.132	7.562
12%	22	12.100	0.083	92.503	0.0108	0.131	7.645
12%	23	13.552	0.074	104.603	0.0096	0.130	7.718
12%	24	15.179	0.066	118.155	0.0085	0.128	7.784
12%	25	17.000	0.059	133.334	0.0075	0.127	7.843
12%	26	19.040	0.053	150.334	0.0067	0.127	7.896
12%	27	21.325	0.047	169.374	0.0059	0.126	7.943
12%	28	23.884	0.042	190.699	0.0052	0.125	7.984
12%	29	26.750	0.037	214.583	0.0047	0.125	8.022
12%	30	29.960	0.033	241.333	0.0041	0.124	8.055
12%	35	52.800	0.019	431.663	0.0023	0.122	8.176
12%	40	93.051	0.011	767.091	0.0013	0.121	8.244
12%	45	163.988	0.006	1358.23	0.0007	0.121	8.283
12%	50	289.002	0.003	2400.02	0.0004	0.120	8.304
12%	55	509.321	0.002	4236.01	0.0002	0.120	8.317
12%	60	897.597	0.001	7471.64	0.0001	0.120	8.324
12%	65	1581.87	0.001	13173.94	0.0001	0.120	8.328
12%	70	2787.80	0.000	23223.33	0.0000	0.120	8.330
12%	75	4913.06	0.000	40933.80	0.0000	0.120	8.332
12%	80	8658.48	0.000	72145.7	0.0000	0.120	8.332
12%	85	15259.21	0.000	127151.7	0.0000	0.120	8.333
12%	90	26891.93	0.000	224091.1	0.0000	0.120	8.333
12%	95	47392.78	0.000	394931.5	0.0000	0.120	8.333
12%	100	83522.27	0.000	696010.5	0.0000	0.120	8.333

Period (years)	**n**
Rate (%)	**i**
Present value ($)	**P**
Future value($)	**F**
Uniform Annual amount	**A**

Single Payment Compound Interest factor	**F/Pi,n**
Present worth factor	**P/Fi,n**
Uniform series Compound amount factor	**F/Ai,n**
Sinking fund factor	**A/Fi,n**
Capital Recovery factor (mortgage)	**A/Pi,n**
Uniform series Present worth factor	**P/Ai,n**

$$P/Fi,n = \frac{1}{F/Pi,n}$$

Compounding Interest Table
i = 10%

i = Rate %	n years	F/Pi,n	P/Fi,n	F/Ai,n	A/Fi,n	A/Pi,n	P/Ai,n
10%	0	1.000	1.000	0.000			
10%	1	1.100	0.909	1.000	1.0000	1.100	0.909
10%	2	1.210	0.826	2.100	0.4762	0.576	1.736
10%	3	1.331	0.751	3.310	0.3021	0.402	2.487
10%	4	1.464	0.683	4.641	0.2155	0.315	3.170
10%	5	1.611	0.621	6.105	0.1638	0.264	3.791
10%	6	1.772	0.564	7.716	0.1296	0.230	4.355
10%	7	1.949	0.513	9.487	0.1054	0.205	4.868
10%	8	2.144	0.467	11.436	0.0874	0.187	5.335
10%	9	2.358	0.424	13.579	0.0736	0.174	5.759
10%	10	2.594	0.386	15.937	0.0627	0.163	6.145
10%	11	2.853	0.350	18.531	0.0540	0.154	6.495
10%	12	3.138	0.319	21.384	0.0468	0.147	6.814
10%	13	3.452	0.290	24.523	0.0408	0.141	7.103
10%	14	3.797	0.263	27.975	0.0357	0.136	7.367
10%	15	4.177	0.239	31.772	0.0315	0.131	7.606
10%	16	4.595	0.218	35.950	0.0278	0.128	7.824
10%	17	5.054	0.198	40.545	0.0247	0.125	8.022
10%	18	5.560	0.180	45.599	0.0219	0.122	8.201
10%	19	6.116	0.164	51.159	0.0195	0.120	8.365
10%	20	6.727	0.149	57.275	0.0175	0.117	8.514
10%	21	7.400	0.135	64.002	0.0156	0.116	8.649
10%	22	8.140	0.123	71.403	0.0140	0.114	8.772
10%	23	8.954	0.112	79.543	0.0126	0.113	8.883
10%	24	9.850	0.102	88.497	0.0113	0.111	8.985
10%	25	10.835	0.092	98.347	0.0102	0.110	9.077
10%	26	11.918	0.084	109.182	0.0092	0.109	9.161
10%	27	13.110	0.076	121.100	0.0083	0.108	9.237
10%	28	14.421	0.069	134.210	0.0075	0.107	9.307
10%	29	15.863	0.063	148.631	0.0067	0.107	9.370
10%	30	17.449	0.057	164.494	0.0061	0.106	9.427
10%	35	28.102	0.036	271.024	0.0037	0.104	9.644
10%	40	45.259	0.022	442.593	0.0023	0.102	9.779
10%	45	72.890	0.014	718.90	0.0014	0.101	9.863
10%	50	117.391	0.009	1163.91	0.0009	0.101	9.915
10%	55	189.059	0.005	1880.59	0.0005	0.101	9.947
10%	60	304.482	0.003	3034.82	0.0003	0.100	9.967
10%	65	490.37	0.002	4893.71	0.0002	0.100	9.980
10%	70	789.75	0.001	7887.47	0.0001	0.100	9.987
10%	75	1271.90	0.001	12708.95	0.0001	0.100	9.992
10%	80	2048.40	0.000	20474.0	0.0000	0.100	9.995
10%	85	3298.97	0.000	32979.7	0.0000	0.100	9.997
10%	90	5313.02	0.000	53120.2	0.0000	0.100	9.998
10%	95	8556.68	0.000	85556.8	0.0000	0.100	9.999
10%	100	13780.61	0.000	137796.1	0.0000	0.100	9.999

Period (years)	**n**
Rate (%)	**i**
Present value ($)	**P**
Future value($)	**F**
Uniform Annual amount	**A**

Single Payment Compound Interest factor	**F/Pi,n**
Present worth factor	**P/Fi,n**
Uniform series Compound amount factor	**F/Ai,n**
Sinking fund factor	**A/Fi,n**
Capital Recovery factor (mortgage)	**A/Pi,n**
Uniform series Present worth factor	**P/Ai,n**

$$P/F_{i,n} = \frac{1}{F/P_{i,n}}$$

Compounding Interest Table
i = 15%

i = Rate %	n years	F/Pi,n	P/Fi,n	F/Ai,n	A/Fi,n	A/Pi,n	P/Ai,n
15%	0	1.000	1.000	0.000			
15%	1	1.150	0.870	1.000	1.0000	1.150	0.870
15%	2	1.323	0.756	2.150	0.4651	0.615	1.626
15%	3	1.521	0.658	3.472	0.2880	0.438	2.283
15%	4	1.749	0.572	4.993	0.2003	0.350	2.855
15%	5	2.011	0.497	6.742	0.1483	0.298	3.352
15%	6	2.313	0.432	8.754	0.1142	0.264	3.784
15%	7	2.660	0.376	11.067	0.0904	0.240	4.160
15%	8	3.059	0.327	13.727	0.0729	0.223	4.487
15%	9	3.518	0.284	16.786	0.0596	0.210	4.772
15%	10	4.046	0.247	20.304	0.0493	0.199	5.019
15%	11	4.652	0.215	24.349	0.0411	0.191	5.234
15%	12	5.350	0.187	29.002	0.0345	0.184	5.421
15%	13	6.153	0.163	34.352	0.0291	0.179	5.583
15%	14	7.076	0.141	40.505	0.0247	0.175	5.724
15%	15	8.137	0.123	47.580	0.0210	0.171	5.847
15%	16	9.358	0.107	55.717	0.0179	0.168	5.954
15%	17	10.761	0.093	65.075	0.0154	0.165	6.047
15%	18	12.375	0.081	75.836	0.0132	0.163	6.128
15%	19	14.232	0.070	88.212	0.0113	0.161	6.198
15%	20	16.367	0.061	102.444	0.0098	0.160	6.259
15%	21	18.822	0.053	118.810	0.0084	0.158	6.312
15%	22	21.645	0.046	137.632	0.0073	0.157	6.359
15%	23	24.891	0.040	159.276	0.0063	0.156	6.399
15%	24	28.625	0.035	184.168	0.0054	0.155	6.434
15%	25	32.919	0.030	212.793	0.0047	0.155	6.464
15%	26	37.857	0.026	245.712	0.0041	0.154	6.491
15%	27	43.535	0.023	283.569	0.0035	0.154	6.514
15%	28	50.066	0.020	327.104	0.0031	0.153	6.534
15%	29	57.575	0.017	377.170	0.0027	0.153	6.551
15%	30	66.212	0.015	434.745	0.0023	0.152	6.566
15%	35	133.176	0.008	881.170	0.0011	0.151	6.617
15%	40	267.864	0.004	1779.090	0.0006	0.151	6.642
15%	45	538.769	0.002	3585.13	0.0003	0.150	6.654
15%	50	1083.657	0.001	7217.72	0.0001	0.150	6.661
15%	55	2179.622	0.000	14524.15	0.0001	0.150	6.664
15%	60	4383.999	0.000	29219.99	0.0000	0.150	6.665
15%	65	8817.79	0.000	58778.58	0.0000	0.150	6.666
15%	70	17735.72	0.000	118231.47	0.0000	0.150	6.666
15%	75	35672.87	0.000	237812.45	0.0000	0.150	6.666
15%	80	71750.88	0.000	478332.5	0.0000	0.150	6.667
15%	85	144316.65	0.000	962104.3	0.0000	0.150	6.667
15%	90	290272.33	0.000	1935142.2	0.0000	0.150	6.667
15%	95	583841.33	0.000	3892268.9	0.0000	0.150	6.667
15%	100	**************	0.000	7828749.7	0.0000	0.150	6.667

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$$P/Fi,n = \frac{1}{F/Pi,n}$$